## Day 1: Identities

## Day 2: Inverses of Trig Functions

## Day 3: Solving Trig Equations

## Day 4: Right Triangles & Trig Ratios: Find Lengths of Sides

## Day 5: Right Triangles & Inverse Trig Ratios: Find Lengths of Angles

## Day 6: More applications of right triangle trigonometry

## Day 7: Bearing Problems

## Day 8: Review
A trigonometric ______________ is a trig equation that is true for all values except those for which the expression on either side of the equal sign are undefined.

We already know a couple of trig identities. For example, we know \( \tan \theta = \)

We also know the reciprocal identity, \( \cot \theta = \)

Let’s use these two identities to derive the cotangent identity for \( \cot \theta = \frac{1}{\tan \theta} = \)

Thus, the \( \cot \theta = \)

We’ve also learned two other reciprocal identities: \( \csc \theta = \) and \( \sec \theta = \)

In addition, there are three Pythagorean Trig Identities.

Derive the following: \( \cos^2 \theta + \sin^2 \theta = 1. \)

Let’s prove the other two Pythagorean Identities by “verifying.”

**Verifying identities:**
To verify an identity, you should transform one side of the equation until it is the same as the other side. This eliminates the possibility of introducing errors that can be caused by squaring both sides of an equation or multiplying both sides of an equation by an expression that equals 0. These are the errors that can introduce extraneous solutions when solving equations.

1. Verify the Pythagorean identity:
   \( 1 + \tan^2 \theta = \sec^2 \theta \)

2. Verify the Pythagorean Identity
   \( 1 + \cot^2 \theta = \csc^2 \theta \)
## Summary of Trigonometric Identities

<table>
<thead>
<tr>
<th></th>
<th>( \csc \theta = )</th>
<th>( \sec \theta = )</th>
<th>( \cot \theta = )</th>
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</thead>
<tbody>
<tr>
<td><strong>Reciprocal</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tangent &amp; Cotangent</strong></td>
<td>( \tan \theta = )</td>
<td></td>
<td>( \cot \theta = )</td>
</tr>
<tr>
<td><strong>Pythagorean</strong></td>
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</table>

2. Verify the identity: \( \tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta \)

3. Verify the identity \( \sec^2 \theta - \sec^2 \theta \cos^2 \theta = \tan^2 \theta \)

4. Simplify the trig expression \( \csc \theta \tan \theta \)

5. Simplify the trig expression \( \sec \theta \cot \theta \)
**U14D2: Inverses of Trig Identities**

Warmup: Verify each identity...

1. \( \sin \theta \sec \theta \cot \theta = 1 \)

2. \( (1 - \sin \theta)(1 + \sin \theta) = \cos^2 \theta \)

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We have spent a lot of time answering questions like \( \sin 135^\circ = \) ________.

Today we are going to work backwards... \( \sin \) ________ \( ^\circ \) = \( \frac{1}{2} \)

When you are trying to find the angle of a trig function, you must use the _______________!!

In other words, ____________________________.

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### Finding Angles Using Trig Functions

<table>
<thead>
<tr>
<th>Finding Angles Using Trig Functions</th>
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</thead>
<tbody>
<tr>
<td>There is a “nice” answer:</td>
</tr>
<tr>
<td>Example: Find an angle whose cosine = ( -\frac{\sqrt{2}}{2} ) (note: find both, in degree)</td>
</tr>
</tbody>
</table>

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### Why there are two answers...

<table>
<thead>
<tr>
<th>The Cosine</th>
<th>The Sine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>Negative</td>
<td>Negative</td>
</tr>
</tbody>
</table>

Note: There are an infinite number of angles. Take “Answer + n \cdot 360^\circ” or “Answer + n \cdot 2\pi”

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Examples: Use the unit circle to find the degree measures of the angles whose cosine is:

a. \( \frac{1}{2} \)

b. \( -\frac{1}{2} \)

c. \( -\frac{\sqrt{3}}{2} \)

d. \( \frac{\sqrt{2}}{2} \)
Now we will find measures using the graphing calculator.

- Find the first angle by _____________________________ (make sure the mode is right!)
- Find the second angle by using the _______________ angle (make sure quadrant is right!)
- Find all of the rest of the angles by adding __________ (degree) or __________ (rad)

### Finding Reference Angles When Angle is in...

<table>
<thead>
<tr>
<th>Quadrant I</th>
<th>Quadrant II</th>
<th>Quadrant III</th>
<th>Quadrant IV</th>
</tr>
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<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
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**Note:** When calc gives neg. angle, ref angle is abs value.

### Once you have the Reference Angle, Find 2nd Angle by...

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2. Use a calculator to find the inverse in **radian** measures of all angles whose:

a. \( \sin \theta = (-.9) \)  
b. \( \sin \theta = 0.44 \)  
c. \( \sin \theta = -0.73 \)
3. Use a calculator and an inverse function to find the measure in \textbf{radians} of all the angles whose tangent is:
   a. -0.84  
   b. 0.44  
   c. -0.73  

*** What is the domain for \( y = \sin^{-1}(x) \)? ***

4. Use a calculator and the inverse functions to find the radian measures of the angles.
   a. Angles whose tangent is 2.5
   b. Angles whose sine is 0.75
   c. Angles whose cosine is (-0.24)
   d. Angles whose cosine is (0.45)

5. Use a unit circle to find the degree measures of the angles.
   a. Angles whose sine is \( \frac{\sqrt{2}}{2} \)
   b. Angles whose tangent is 1
   c. Angles whose cosine is \( \frac{\sqrt{2}}{2} \)
   d. Angle whose sine is 1
**U14D3: Solving Trig Equations**

Warmup: 1) Find the radian measure of an angle between 0 and $2\pi$ of $\tan^{-1}(2.5)$

2) Solve the equation $x^2 - x - 20 = 0$

To solve trig equations, follow these steps…

- If there is only one function, isolate that function and then take the inverse (like yesterday)
- If there are multiple trig functions, factor and use the zero product property – then take inverses!

**Directions:** Solve each equation for $0 \leq \theta \leq 2\pi$ (make sure your answers are in the domain!!!)

1. $2 \cos \theta - \sqrt{3} = 0$

2. $2 \sin \theta - \sqrt{2} = 0$

3. $3 \tan \theta - 1 = \tan \theta$

4. $2 \cos \theta \sin \theta + \sin \theta = 0$

5. $(\sin \theta - 1)(\sin \theta + 1) = 0$

6. $3 \tan \theta + 5 = 0$
Flashback: You already know how to find a missing side of a right triangle, given 2 sides:

Write the equation and the name of the Theorem you would use.

Special Right Triangles
And you know how to find the missing side if the triangle is a 30-60-90 or a 45-45-90 right triangle.

Fill in the blanks.

But what if you don’t know the angles measures or the angle measures are not one of the special triangles?

4. In a right triangle, that has an acute angle $A$, the ratios are defined as follows:

$$\sin \angle A = \frac{opp}{hyp} \quad \cos \angle A = \frac{adj}{hyp} \quad \tan \angle A = \frac{opp}{adj}$$

opp = side opposite the angle \quad adj = side adjacent the angle \quad hyp = hypotenuse
5. Based on what you know about the reciprocal trig functions (csc $\theta$, sec $\theta$, and cot $\theta$) what do you think the ratios are for these functions?

6. A little proof for you. For acute angles, the unit circle definition of sine is equivalent to the definition of sine for right triangles.

\[ \sin \theta = y\text{-coordinate of } P = PQ \]

\[ \sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} \]

Since $\triangle APQ$ and $\triangle ABC$ are similar triangles, \[ \frac{PQ}{PA} = \frac{BC}{AB} \]

So, \[ \sin \theta = PQ = \frac{PQ}{1} = \frac{PQ}{PA} = \frac{BC}{AB} = \sin A \]

This would hold true for the other 5 trig ratios.
6. From the triangle below, find all 6 trig function values for both angle A and angle B.

\[ \sin A = \_ \_ \_ \quad \cos B = \_ \_ \_ \]
\[ \cos A = \_ \_ \_ \quad \sin B = \_ \_ \_ \]
\[ \tan A = \_ \_ \_ \quad \cot B = \_ \_ \_ \]
\[ \cot A = \_ \_ \_ \quad \tan B = \_ \_ \_ \]
\[ \sec A = \_ \_ \_ \quad \csc B = \_ \_ \_ \]
\[ \csc A = \_ \_ \_ \quad \sec B = \_ \_ \_ \]

8. What do you notice? Why does this make sense? What is the relationship between angle A and angle B (think back to the last unit. Doesn’t sin 30° = cos 60°?)

9. The \( \tan E = \frac{1}{4} = \frac{\text{side opposite } E}{\text{side adjacent to } E} \) in \( \triangle DEF \).

Use the Pythagorean Theorem to find the length of the hypotenuse. Then, find the values of \( \sin E \) and \( \sec F \) in fraction form.
10. Use the right triangle definitions to find \( c \) in \( \Delta ABC \) for each case described below. The first is solved for you. (Use your calculator in degree mode) Round values to the nearest hundredth.

a.

\[
\cos 33^\circ = \frac{c}{7}
\]

therefore we can write:

\[
7 \cos 33 = c
\]

\[
c \approx 5.87
\]

b.

Find the value of \( c \).

c.

Find the value of \( c \). After finding that side length, use the Pythagorean Theorem to find the length of the remaining side.

d.

Find the value of \( c \).
To find the measures of an acute angle in right triangle, you can use the inverses of the trigonometric functions.

Remember:  \[ \sin(\ ) = \quad \text{&} \quad \sin^{-1}(\ ) = \quad \]

11. Use \( \triangle DEF \) to find the measure of each angle to the nearest tenth of a degree.

12. \( x = \quad \)

\( y = \quad \)

13. \( x = \quad \)

\( y = \quad \)
14. A wheelchair ramp must be constructed so the slope is not more than 1 in. of rise for every 1 ft of run. What is the maximum angle that the ramp can make with the ground, to the nearest tenth of a degree?

![Diagram of a ramp with angle θ and rise 1 inch over run 1 foot.]

15. A straight road that goes up a hill is 800 feet higher at the top than at the bottom. The horizontal distance covered is 6515 feet. To the nearest degree, what angle does the road make with level ground?

16. Park planners would like to build a bridge across a creek. Surveyors have determined that from 5 ft above ground the angle of elevation to the top of an 8 foot pole on the opposite side of the creek is 5°. Find the length of the bridge to the nearest foot.

![Diagram of a bridge with angle of elevation 5°, rise 5 feet, and run unknown. The pole is 8 feet.]
Working with Degrees, Minutes, and Seconds

There are several ways to measure the size of an angle. One way is to use units of degrees. (Radian measure is another way.)

In a complete circle there are three hundred and sixty degrees.

An angle could have a measurement of 35.75 degrees. That is, the size of the angle in this case would be thirty-five full degrees plus seventy-five hundredths, or three fourths, of an additional degree. Notice that here we are expressing the measurement as a decimal number. Using decimal numbers like this one can express angles to any precision - to hundredths of a degree, to thousandths of a degree, and so on.

There is another way to state the size of an angle, one that subdivides a degree using a system different than the decimal number example given above. The degree is divided into sixty parts called minutes. These minutes are further divided into sixty parts called seconds. The words minute and second used in this context have no immediate connection to how those words are usually used as amounts of time.

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In a full circle there are 360 degrees.

Each degree is split up into 60 parts, each part being 1/60 of a degree. These parts are called minutes.

Each minute is split up into 60 parts, each part being 1/60 of a minute. These parts are called seconds.

The size of an angle could be stated this way: 40 degrees, 20 minutes, 50 seconds.

There are symbols that are used when stating angles using degrees, minutes, and seconds. Those symbols are show in the following table.

So, the angle of 40 degrees, 20 minutes, 50 seconds is usually written this way:

\[ 40^\circ 20'50'' \]
How could you state the previous angle as an angle using common decimal notation? The angle would be this many degrees, (* means times.):

\[ 40 + (20 \times \frac{1}{60}) + (50 \times \frac{1}{60} \times \frac{1}{60}) \]

That is, we have 40 full degrees, 20 minutes - each 1/60 of a degree, and 50 seconds - each 1/60 of 1/60 of a degree.

Work that out and you will get a decimal number of degrees. It's 40.34722...

Going the other way is a bit more difficult. Suppose we start with 40.3472 degrees. Can we express that in units of degrees, minutes, and seconds?

Well, first of all there are definitely 40 degrees full degrees. That leaves 0.3472 degrees.

So, how many minutes is 0.3472 degrees? Well, how many times can \( \frac{1}{60} \) go into 0.3472? Here's the same question: What is 60 times 0.3472? It's 20.832. So, there are 20 complete minutes with 0.832 of a minute remaining.

How many seconds are in the last 0.832 minutes. Well, how many times can \( \frac{1}{60} \) go into 0.832, or what is 60 times 0.832? It's 49.92, or almost 50 seconds.

So, we've figured that 40.3472 degrees is almost exactly equal to 40 degrees, 20 minutes, 50 seconds.

(The only reason we fell a bit short of 50 seconds is that we really used a slightly smaller angle in this second half of the calculation explanation. In the original angle, 40.34722... degrees, the decimal repeats the last digit of 2

Adapted from: [http://id.mind.net/~zona/mmts/trigonometryRealms/degMinSec/degMinSec.htm](http://id.mind.net/~zona/mmts/trigonometryRealms/degMinSec/degMinSec.htm)

17. Practice converting from degrees to minute/second notation

   a. convert 38.296 to degree-minute-second notation

   b. convert 28°37'51" to degree notation

Using your Calculator:
   (you will like this)
18. Michelle knows that when she stands 123 feet from the base of a flagpole, the angle of elevation to the top is 26 degrees, 40 minutes. If her eyes are 5.3 feet above the ground, find the height of the flagpole.

19. The length of the shadow of a building 34.09 meters tall is 37.62 meters. Find the angle of elevation of the sun.

20. Suppose you are standing on one bank of a river. A tree on the other side of the river is known to be 150 feet tall. A line from the top of the tree to the ground at your feet makes an angle of 11° with the ground. How far from you is the base of the tree?

21. A kite string makes a 62° angle with the horizontal, and 300 ft of string is let out. The string is held 6 ft. off the ground. How high is the kite?

22. A man 6 feet tall is standing 50 feet from a tree. When he looks at the top of the tree, the angle of elevation is 42°. Find the height of the tree to the nearest foot.

23. A BRIEF intro to bearing problems:
   a. Used in ________________________
   b. When a single angle is given, it is understood that the bearing is measured in a clockwise direction from due north.
1. The angle of elevation from a point on the ground to the top of a tower is 35°.
   From a point 55ft closer to the tower, the angle of elevation is 48°. How tall is the tower?

2. A math class is finding the height of a tree using trigonometry. They measure the angle of
elevation from a point on the ground to be 36°20'. When they move back 40ft, they measure
the angle of elevation to be 21°50'. What is the height of the tree?

3. A weather station is watching the linear movement of a tornado as it approaches a tower.
   From the top of the tower, the angle of depression is 25°30'. Just before the twister dies, the
   angle of depression changes to 48°40'. If the top of the tower is 50ft above the top of the
   twister, how far did the tornado travel along the ground as it approached the tower?

4. The height of an observation tower is 200ft. Tourist A is on the ground looking up at the
tower. Tourist B is 75ft behind tourist A, and looking up at the tower with an angle of
elevation of 38°10'. Find the angle of elevation from tourist A to the top of the tower.
**U14D7: Bearing Problems**

Two types of bearings:

1) Bearing in relation to the North
   Ex: Bearing of 220°

2) Bearing given for a direction
   Ex: Bearing of S 75°W

1. Two radio towers are on a north-south line. From Tower A, a ship is located at a bearing of 160°. From tower B, the ship is located at the bearing of 70°. If the towers are 100 miles from each other, how far is the ship from each tower?

![Radio Tower]

2. A plane flies between 3 cities. The bearing are:

   From A to B N 61° E
   From B to C S 19° W
   From C to A N 29° W

   If the distance from A to B is 150 miles, find the distances from B to C and C to A.

![Plane]
Bearing Problems:

5. Two towers are on a north-south line. From tower A, a ship is located at a bearing of 247°. From tower B, the ship is located at a bearing of 337°. Find the distance between the towers if the distance from tower A to the ship is 2572 meters.

6. Two lighthouses are on an east-west line. From lighthouse C, a boat in distress is 15.4 miles away with a bearing of 328°. From lighthouse D, the boat has a bearing of 58°. How far will a rescue boat have to travel from D to get to the boat?

7. Radar stations A and B are on an east-west line, 3.7km apart. Station A detects a plane at C, with a bearing of 61°. Station B also detects the plane with a bearing of 331°. Find the distance from A to C.

8. The bearing from point X to point Z is S 52 E. The bearing from X to Y is N 84 E. The bearing from Y to Z is S 38 W. If the distance from X to Z is 50m, find the distance from X to Y.

9. The corners of a triangular piece of land are located on a map as points R, S and T. From R, point S is N 26 W. From S, point T is S 64 W. If RS is 528 ft, and RT is 945 ft, find \( m \angle T \) and the distance from S to T.