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Warmup: Quick review of graphing calculator procedures

Day 1 notes are on your own, based on the Powerpoint – here are some things you need to know:

- Organize data into a matrix
- Identify the size of a matrix (Row X Column, not the other way around!)
- Identify elements in matrix according to naming conventions
- Add and subtract matrices, and solve equations using adding and subtracting
- Determining equal matrices – and solving equations based on equal matrices
- Scalar multiplication
- Matrix multiplication – know how to do it, and when it is defined
A ____________________ is a matrix with the same number of columns as rows.

**Identity matrix**: A matrix with 1’s down the diagonal (an identity is called “I”)

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\quad \text{2 x 2 Identify} \quad \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\quad \text{3 x 3 Identify}
\]

If the product of the real numbers \(a\) and \(b = 1\), then \(a\) and \(b\) are ___________________ ________________ (reciprocals). This same logic can be applied to matrices…except not all matrices have inverses.

**Multiplicative Inverse**: If \(A\) and \(X\) are \(n \times n\) matrices (square), and \(AX =XA = I\), then \(X\) is the multiplicative inverse of \(A\) and is written \(A^{-1}\).

SO \(AA^{-1} = A^{-1}A = I\)

Examples:

1. Show that \(B\) is the multiplicative inverse of \(A\).

\[
A = \begin{bmatrix}
2 & 3 \\
1 & 2
\end{bmatrix} \quad B = \begin{bmatrix}
2 & -3 \\
-1 & 2
\end{bmatrix}
\]

2. Show that \(B\) is the multiplicative inverse of \(A\)

\[
A = \begin{bmatrix}
-2 & -5 \\
-3 & -8
\end{bmatrix} \quad B = \begin{bmatrix}
-8 & 5 \\
3 & -2
\end{bmatrix}
\]

The determinant of a 2 x 2 matrix is defined as follows:
If \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), then \( \det(A) = ad - bc \)

Often times, instead of writing \( \det(A) \), we will write \( \begin{vmatrix} a & b \\ c & d \end{vmatrix} \).

The straight bars of a matrix represent the “determinant”

\[
A = \begin{bmatrix} -1 & 7 \\ 2 & -3 \end{bmatrix}, \text{ find } |A|.
\]

So, \( \begin{vmatrix} -1 & 7 \\ 2 & -3 \end{vmatrix} = (-1)(-3) - (7)(2) \)

Thus, \( \det(A) = 3 - 14 = -11 \)

Evaluate each determinant matrix. Determinant notation \( \Rightarrow \)

1. \( \begin{bmatrix} -3 & 4 \\ 2 & -5 \end{bmatrix} \)

2. \( \begin{bmatrix} -2 & 2 \\ 3 & -4 \end{bmatrix} \)

3. \( \begin{bmatrix} 3 & 3 \\ 3-k & -3 \end{bmatrix} \)

**Calculator Notes:**

**Inverses: Powerpoint Demonstration**

Formally stated: If \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \), …

1. Find the inverse of \( A = \begin{bmatrix} -2 & 2 \\ 5 & -4 \end{bmatrix} \)
2. Find the inverse of \( A = \begin{bmatrix} .5 & 2.3 \\ 3 & 7.2 \end{bmatrix} \)

Use the inverse of a determinant to solve a matrix equation \( AX = B \)

Normally, if there were a typical algebra problem, you would divide by \( A \) to solve for \( x \). However, we don’t have matrix division, just inverse matrices (essentially the same thing). So, you have to take the inverse of \( A \) and multiply both sides by \( A^{-1} \). Remember this will give you the identity matrix on the left, which is equivalent to 1 in “typical” algebra.

1. Solve \( \begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \)

2. Solve \( \begin{bmatrix} 3 & -4 \\ 4 & -5 \end{bmatrix} X = \begin{bmatrix} 0 & -22 \\ 0 & -28 \end{bmatrix} \)

3. Solve \( \begin{bmatrix} 4 & 7 \\ 1 & x \end{bmatrix} X + \begin{bmatrix} 2 & 7 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ -2 & 3 \end{bmatrix} \)

Closure: When does an inverse not exist? And what is an inverse matrix used for?
Method 1: By Minors

Step 1: Circle the entry in the top left corner. Then eliminate the rest of the row and column. 

\[
\begin{pmatrix}
1 & 2 & -1 \\
3 & 0 & 4 \\
6 & 1 & 2 \\
\end{pmatrix}
\]

Step 2: Take the number in the circle, times the determinant of the 2 x 2

\[
= 1 \begin{vmatrix}
0 & 4 \\
1 & 2 \\
\end{vmatrix}
= 1[(0)(2) - (4)(1)] = 1[-4] = -4
\]

Step 3: Now move the circle to the right. Eliminate the rest of this row and column

\[
\begin{pmatrix}
1 & 2 & -1 \\
3 & 0 & 4 \\
6 & 1 & 2 \\
\end{pmatrix}
\]

Step 4: Take the OPPOSITE of number in the circle, times the determinant

\[
-2 \begin{vmatrix}
3 & 4 \\
6 & 2 \\
\end{vmatrix}
\]

Step 5: Now move the circle to the right again. Eliminate the rest of this row and column.

\[
\begin{pmatrix}
1 & 2 & -1 \\
3 & 0 & 4 \\
6 & 1 & 2 \\
\end{pmatrix}
\]

Step 6: Take the number in the circle, times the determinant again.

\[
-1 \begin{vmatrix}
3 & 0 \\
6 & 1 \\
\end{vmatrix}
= -1[(3)(1) - (0)(6)] = -1[3 - 0] = -1[3] = -3
\]

Final Step: Add the 3 values together.

\[
\text{Determinant} = -4 + 36 + -3 = 29
\]

Note: you can use any row (or column). Just be sure to follow this pattern:
Sometimes it will be be easier (for example, one row has 2 zeros)

+ - +  
- + -  
+ - +
Method 2: Using Diagonals

Step 1: Write the matrix twice, right next to each other.

\[
\begin{bmatrix}
1 & 2 & -1 \\
3 & 0 & 4 \\
6 & 1 & 2 \\
\end{bmatrix}
\begin{bmatrix}
1 & 2 & -1 \\
3 & 0 & 4 \\
6 & 1 & 2 \\
\end{bmatrix}
\]

Step 2: Draw 3 diagonal ovals from the top row of the first matrix, as shown.

\[
\begin{bmatrix}
1 & 2 & -1 & 1 & 2 & -1 \\
3 & 0 & 4 & 3 & 0 & 4 \\
6 & 1 & 2 & 6 & 1 & 2 \\
\end{bmatrix}
\]

Multiply the 3 numbers in each oval individually:

\[
(1)(0)(2) + (2)(4)(6) + (-1)(3)(1) = 0 + 48 - 3 = \text{This} = 45
\]

So this part = 45

Step 3: Now draw the ovals going up, starting from the bottom...

\[
\begin{bmatrix}
1 & 2 & -1 & 1 & 2 & -1 \\
3 & 0 & 4 & 3 & 0 & 4 \\
6 & 1 & 2 & 6 & 1 & 2 \\
\end{bmatrix}
\]

Again, multiply all the ovals. But this time, take the OPPOSITE sign for each piece!

So this equals:

\[
-(6)(0)(-1) - (1)(4)(1) - (2)(3)(2) = +0 - 4 - 12 = -16
\]

Final Step: Find the determinant by adding the two pieces together.

\[
\text{Again, determinant } 45 + (-16) = 29
\]

Thinking Skill: C-8: Examine Information from more than one point of view
Objective: Use Cramer’s Rule to Solve Systems

Thinking Skill: C-3: Gather and organize information and data

\[ D = \begin{vmatrix} x & y \\ c & c \\ o & o \\ e & e \\ f & f \end{vmatrix} \quad D_x = \begin{vmatrix} c & y \\ c & c \\ o & o \\ s & t \\ e & f \end{vmatrix} \quad D_y = \begin{vmatrix} x & c \\ c & c \\ o & o \\ s & e \\ t & f \end{vmatrix} \]

* Note: The “constants” column always goes in place of the subscript letter’s column

2. Cramer's Rule - use the example above to solve the system below using Cramer's Rule.

\[2x + 3y = 8\]
\[3x - y = 1\]

\[ D = \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = \]

\[ D_x = \begin{vmatrix} 3 & 3 \\ 3 & -1 \end{vmatrix} = \]

* constants go first!

\[ D_y = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = \]

\[ x = \quad y = \]
Applying Cramer’s Rule to 3 x 3 Matrices ➔ The four columns are: \( x \ y \ z \ c \)

\[
D = \begin{vmatrix} x & y & z \end{vmatrix}, \quad
D_x = \begin{vmatrix} 1 & y & z \end{vmatrix}, \quad
D_y = \begin{vmatrix} x & 1 & z \end{vmatrix}, \quad
D_z = \begin{vmatrix} x & y & 1 \end{vmatrix}
\]

Try the following:

1. \[
\begin{align*}
2x + y &= 1 \\
3x - y &= 9
\end{align*}
\]

2. \[
\begin{align*}
x + y + z &= 0 \\
2x - 2y + 3z &= 46 \\
3x + 7y + 11z &= 80
\end{align*}
\]

3. \[
\begin{align*}
3x + y + z &= 18 \\
4x + 2y + 3z &= 12 \\
7x + 8y + 5z &= 9
\end{align*}
\]

Solve for \( z \) only!

4. \[
\begin{align*}
3x + 5y &= 1 \\
x + 6y &= 9
\end{align*}
\]

Solve for \( y \) only!
Suppose you invested $5000 in three different mutual funds for one year. The funds paid simple interest of 8%, 10%, and 7%, respectively. The total interest at the end of the one year was $405. You invested $500 more at 10% than at 8%. How much did you invest in each mutual fund?

Closure: How can you remember the order for Cramer’s Rule? Why would it be helpful (compared to the other methods for solving systems that we have learned)?
Thinking Skill: C-5: Demonstrate and understanding of concepts

Find the inverse of each 2x2 matrix.

1. \[
\begin{bmatrix}
2 & -1 \\
3 & 1
\end{bmatrix}
\]
2. \[
\begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
-3 & 4 \\
1 & -2
\end{bmatrix}
\]

System of linear equations

\[
\begin{align*}
x + 2y &= 5 \\
3x + 5y &= 14
\end{align*}
\]
Matrix Equation

\[
\begin{bmatrix}
1 & 2 \\
3 & 5
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
5 \\
14
\end{bmatrix}
\]

So, \( AX = B \Rightarrow \)

Examples: Write the following system as a matrix equation

1. \[
\begin{align*}
-b + 2c &= 4 \\
a + b - c &= 0 \\
2a + 3c &= 11
\end{align*}
\]
2. \[
\begin{align*}
x - y + z &= 0 \\
x - 2y - z &= 5 \\
2x - y + 2z &= 8
\end{align*}
\]

Solving a system of two equations. Remember that a determinant CANNOT be equal to “0”

1. \[
\begin{align*}
2x + 3y &= 11 \\
x + 2y &= 6
\end{align*}
\]
2. \[
\begin{align*}
5a + 3b &= 7 \\
3a + 2b &= 5
\end{align*}
\]
Another Method: Augmented Matrix and RREF!

Note: Your test may require you to use the inverse!

Solving a System with 3 variables using your calculator.

\[ \begin{align*}
2x + y + 3z &= 1 \\
5x + y - 2z &= 8 \\
x - y - 9z &= 5
\end{align*} \]

4. \[ \begin{align*}
x + y + z &= 2 \\
2x + y &= 5 \\
x + 3y - 3z &= 14
\end{align*} \]

6. You want to burn 380 Calories during 40 minutes of exercise. You burn 8 Calories per minute inline skating and 12 Calories per minute swimming. How long should you spend doing each activity?

7. Two friends rent an apartment for $975 per month. Since one bedroom is 60 square feet larger than the other bedroom, each person’s rent contribution is based on bedroom size. Each person agrees to pay $3.25 per square foot of bedroom area. Let x be the area (in square feet) of the larger bedroom, and let y be the area (in square feet) of the smaller bedroom. Write and solve a system of linear equations to find the area of each bedroom.

Closure
Inverses of matrices do not exist when______________________________
1. Your aunt receives an inheritance of $20,000. She wants to put some of the money into a savings account that earns 2% interest annually and invest the rest in certificates of deposit (CDs) and bonds. A broker tells her that CDs pay 5% interest annually and bonds pay 6% interest annually. She wants to earn $1000 interest per year, and she wants to put twice as much money in CDs as in bonds. How much should she put in each type of investment?

2. Jeanette, Raj, and Henry go to a Chinese restaurant for lunch and order 3 different luncheon combination platters. Jeanette orders 2 portions of fried rice and 1 portion of chicken chow mein. Raj orders 1 portion of fried rice, 1 portion of chicken chow mein, and 1 portion of sautéed broccoli. Henry orders 1 portion of sautéed broccoli and 2 portions of chicken chow mein. Janette’s platter cost $5, Raj’s costs $5.25, and Henry’s costs $5.75. How much does 1 portion of chicken chow mein cost?
3. In 1992, there were 548,303 doctors under the age of 65 in the United States. Of those under age 45, 25.53415% were women. Of those between the ages of 45 and 65, 11.67209% were women. There were 110,017 women doctors under the age of 65. How many doctors were under age 45?

4. An apartment building has 50 units. All are one- or two-bedroom units. One-bedroom units rent for $425/month, and two-bedroom units rent for $550/month. When all units are occupied, the total monthly income is $25,000. How many apartments of each type are there?
5. A delicatessen delivers a gigantic sandwich to be shared by the 12 members of a jury who have been unable to reach a verdict. The sandwich consists of bread, meat, and cheese. The cost of the sandwich is $.60/lb for the bread, $3.00/lb for the meat, and $1.50/lb for the cheese. One pound of bread supplies 10 g of protein, one pound of meat supplies 50 g of protein, and a pound of cheese supplies 40 g of protein. Each member of the jury pays $1.50 for a one-pound portion of the sandwich. Each on-pound portion contains 30 g of protein. How much of each ingredient is in the whole sandwich?

6. Paella is a classic Spanish fiesta dish made from chicken, rice, and shellfish. One pound of chicken costs $1.00 and supplies 100 g of protein. One pound of rice costs $.50 and supplies 20 g of protein. One pound of shellfish costs $3.00 and supplies 50 g of protein. If the resulting paella weighs 18 lb., costs $19.00 and supplies 850 g of protein, how much rice, chicken and shellfish were used?
7. The local yogurt bar features a banana treat made up of 2 lb of bananas, 3 lb of topping, and 4 lb of frozen yogurt. The cost of the banana treat is $19.00. One pound of topping costs $1 less than one pound of frozen yogurt, which costs as much as 1/2 lb of topping and 4 lb of bananas. How much does one pound of each ingredient cost?

8. The banana treat in problem 7 contains 5400 calories. There are half as many calories in one pound of frozen yogurt as there are in one pound of topping. Together, 2 lb of frozen yogurt and 5 lb of bananas have the same amount of calories as 2 lb of topping. Find the number of calories in one pound of each of the items.