# Unit 2 Syllabus: Parallel and Perpendicular Lines

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1. Before you leave today, you must know and be able to identify the following terms...
   Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles,
   Same Side Interior Angles, and Same Side Exterior Angles

Warm Up: Read the rest of this page (front and back) and prepare yourself for an activity
involving the terms listed above. You will not be allowed to use your notes during the activity.

2. When two lines are intersected by another line, that line is called a transversal.

3. In the picture above, you should already be able to find vertical angles and supplementary
   angles (straight angles). We will now look at some new terms...

4. Interior Angles – between the 2 lines
   Exterior angles – outside the 2 lines
   (notice the angles are between the 2 parallel lines)                     (notice the angles are outside the 2 parallel lines)

5. Angles on the same side of the transversal are called same-side angles.

6. Angles on opposite sides of the transversal are called alternate angles.
Naming angles involving a transversal...

Same Side Interior Angles

Alternate Interior Angles

Same Side Exterior Angles

Alternate Exterior Angles

7. Angles that “match up” are called corresponding angles. Notice how each line makes a group of 4 angles with the transversal:

* 1a and 1b are corresponding angles – same with 2a & 2b, 3a & 3b, and 4a & 4b.

In other words, “upper left” corresponds with “upper left”
“lower right” corresponds with “lower right,” and so on...
Corresponding Angles Postulate.

If a transversal intersects two parallel lines, then corresponding angles are congruent.

Statement: Lines $l$ and $m$ are parallel, and are intersected by line $q$.
Conclusion:

Visual Representation: Parallel vs. Non Parallel

Do we need to prove this statement? Why or why not?

8. Alternate Interior Angles Theorem (use the postulate above to prove this thm.)

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Given: Parallel Lines $n$ and $m$, with transversal $p$.
Prove: $\angle 3 \cong \angle 6$
9. **Alternate Exterior Angles Theorem**

   If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

   (this is proven in a very similar fashion to the last theorem – try it!)

10. **Same Side Interior Angles Theorem**

   If a transversal intersect two parallel lines, then same side interior angles are supplementary.

   Given: Parallel Lines $n$ and $m$, with transversal $p$.
   
   Prove: $\angle 4 + \angle 6 = 180$

   
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons (Postulates, Theorems, Definitions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle 4 + \angle 6 = 180$</td>
<td></td>
</tr>
</tbody>
</table>

11. **Same Side Exterior Angles Theorem**

   If a transversal intersect two parallel lines, then same side exterior angles are supplementary.

   (this is proven in a very similar fashion to the last theorem – try it!)
1. We ____________ lines are parallel by using the converses of the postulate and theorems we learned yesterday:

These are __________________________ angles.

2. Theorems about the other angles also follow:

1) If alternate interior angles are congruent, then the lines are parallel

Given: \( \angle 3 \cong \angle 6 \)  
Prove: \( m \parallel n \)
2) If alternate exterior angles are congruent, then the lines are parallel.

3) If same-side interior angles are supplementary, then the lines are parallel.

4) If same-side exterior angles are supplementary, then the lines are parallel.

<table>
<thead>
<tr>
<th>Important Information for Proofs!</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Previous Statement</strong></td>
</tr>
<tr>
<td>∠A is a right angle</td>
</tr>
<tr>
<td>∠A and ∠B are rt ∠s</td>
</tr>
<tr>
<td>m ⊥ n</td>
</tr>
<tr>
<td>∠1 + ∠2 = 180</td>
</tr>
</tbody>
</table>

The first **Reason** of a proof is usually “Given”

Corresponding Angles (and its converse) is a **postulate**.

Alternate interior/exterior Angles are congruent and this is a **theorem**.

Same side interior/exterior angles are supplementary and this is also a **theorem**.

If the lines are parallel, and you say something about the angles, then you are using the postulate or theorems

If you know about the angles, then you claim the lines are parallel, you are using the **converses** of the postulate or theorems.
**U2 D2: Proving Lines Parallel**

**Directions:** Determine which lines or segments are parallel and justify your answer with a theorem or postulate.

**Directions:** Find the value of $x$ that will ensure $a \parallel t$.
Directions: Perform the proofs below.

1) \textbf{GIVEN: } k \parallel \ell, \angle 5 \cong \angle 8.
\textbf{PROVE: } j \parallel \ell.

2) \textbf{GIVEN: } \angle K \cong \angle P,
\angle J + m \angle P = 180.
\textbf{PROVE: } \overline{KL} \parallel \overline{JP}.
3. When you have a point and a line...

There is one line that is _____________ to a line through a point.
Called the ________________ _______________

4. If two lines are parallel to a third line, then the lines are parallel to each other...

There is one line that is _____________ to a line through a point.
Called the ________________ _______________

5. If two lines are perpendicular to the same line, then the lines are parallel to each other.
6. Let’s prove the previous theorem.

Given: \( t \perp m \) and \( t \perp n \)

Prove: \( m \parallel n \)

#2 on tonight’s homework (freebie)
1. Triangles can be classified by the number of congruent sides that they have.

<table>
<thead>
<tr>
<th>Zero Congruent Sides</th>
<th>Two+ Congruent Sides</th>
<th>All 3 Congruent Sides</th>
</tr>
</thead>
</table>

2. Triangles can also be classified by their angles.

<table>
<thead>
<tr>
<th>3 Acute Angles</th>
<th>One Obtuse Angle</th>
<th>One Right Angle</th>
<th>All (\cong) Angles</th>
</tr>
</thead>
</table>

3. Always, sometimes, never...
   a. A right triangle is equiangular: _____________________
   b. An acute triangle is equilateral: ____________________
   c. An obtuse triangle is isosceles: _____________________
   d. A right triangle is scalene: _________________________

5. The Triangle Sum Theorem (Δ∑ Thm.)
   a. The sum of the measures of the angles of a triangle is 180°

   \[ m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ \]

   **Proof:**

   Given: Δ ABC with angles 1, 2, and 3

   Prove: \[ m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ \]

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Δ ABC with angles 1, 2, and 3</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Line ( t ) is parallel to ( AB )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( m\angle 4 = m\angle 1 )</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( m\angle 4 + m\angle 3 + m\angle 5 = 180^\circ )</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6. Substitution</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
</tr>
</tbody>
</table>
6. Exterior angles of a triangle – formed by __________________________ one of the sides.

![Diagram of a triangle with exterior angles](image)

What angle is supplementary to that angle?

7. Each exterior angle has two __________________________ interior angles

![Diagram of a triangle](image)

8. Find the degree measure of the exterior angles below...

![Diagram of triangles with exterior angles](image)

9. **Exterior Angles Theorem**: The measure of an exterior angle is equal to...

![Diagram of a triangle with exterior angles](image)
Directions: Find the value of each variable.

1. \[\text{\(x^\circ\)} = \text{\(60^\circ\)}
2. \[\text{\(x^\circ\)} = \text{\(n^\circ\)}
3. \[\text{\(x^\circ\)} = \text{\(93^\circ\)}

Directions: Find the measure of each numbered angle

4. \[\text{\(m^\circ\)}

5. \[\text{\(x^\circ\)} = \text{\(79^\circ\)}

Closure: Compare and contrast the triangle sum theorem and triangle exterior angle thm. How to play “BLUFF!”
2. Polygon means “many angles.” They are closed spatial figures with no curves or overlaps.
3. We almost always deal only with convex polygons – where all the diagonals are inside.

Convex: \[
\text{\includegraphics{convex.png}}
\]
Concave (“cave in”): \[
\text{\includegraphics{concave.png}}
\]

4. Regular polygons have congruent angles (equiangular) and congruent sides (equilateral)

### Formulas for Polygons of \( n \) sides (plug in for \( n \))

<table>
<thead>
<tr>
<th>Sum of Interior Angles</th>
<th>Sum of Exterior Angles</th>
<th>One Interior Angle</th>
<th>One Exterior Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>((n-2) \cdot 180)</td>
<td>360</td>
<td>((n-2) \cdot 180)/(n)</td>
<td>360/(n)</td>
</tr>
</tbody>
</table>

Apply to Regular Polygons ONLY!

Proof (sort of) of Sum of interior angles theorem:

When connecting all of the diagonals of a polygon, it always makes two less triangles than the number of sides (hence the \( n - 2 \)). Each triangle has a sum of 180 degree (already proven).

\[
\begin{align*}
m\angle 1 + m\angle 2 + m\angle 3 &= 180 \\
m\angle 4 + m\angle 5 + m\angle 6 &= 180 \\
m\angle 7 + m\angle 8 + m\angle 9 &= 180
\end{align*}
\]

\[
m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 + m\angle 7 + m\angle 8 + m\angle 9 = 180 + 180 + 180 = 540
\]
Example 1: For a regular decagon (10 sided) find the sum of the interior, sum of the exterior, one interior, and one exterior angle.

Our figure is 10 sided, so $n = 10$. Plug in 10 for the four formulas.

<table>
<thead>
<tr>
<th>Sum of Interior Angles</th>
<th>Sum of Exterior Angles</th>
<th>One Interior Angle</th>
<th>One Exterior Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(n-2)\cdot180$</td>
<td>$360$</td>
<td>$\frac{(n-2)\cdot180}{n}$</td>
<td>$\frac{360}{n}$</td>
</tr>
<tr>
<td>$(10-2)\cdot180$</td>
<td></td>
<td>$\frac{(10-2)\cdot180}{10}$</td>
<td>$\frac{360}{10}$</td>
</tr>
<tr>
<td>$(8)\cdot180$</td>
<td></td>
<td>$\frac{(8)\cdot180}{10}$</td>
<td>$\frac{1440}{10}$</td>
</tr>
<tr>
<td>$1440$</td>
<td></td>
<td>$144$</td>
<td></td>
</tr>
</tbody>
</table>

Example 2: Each interior angle of a regular polygon is 120 degrees. How many sides does it have?

** This problem makes you work BACKWARDS**

You know the answer, now you need to find the question...

**Step 1:** Identify which of the four types of problems you are dealing with. Write that formula

The problem says “Each interior angle” so we want... $\frac{(n-2)\cdot180}{n}$

**Step 2:** Set that formula equal to its value (the “answer”)

$$\frac{(n-2)\cdot180}{n} = 120$$

**Step 3:** Solve for $n$.

$$180n - 360 = 120n$$

Add and Subtract on Both sides

$$60n = 360$$

Divide by 60

$$n = 6$$
1. List the formulas in the appropriate boxes below for a regular polygon of \( n \) sides...

<table>
<thead>
<tr>
<th>Sum of Interior Angles</th>
<th>Sum of Exterior Angles</th>
<th>One Interior Angle</th>
<th>One Exterior Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Find the given angles for a regular 12 sided polygon. SHOW ALL WORK!

<table>
<thead>
<tr>
<th>Sum of Interior Angles</th>
<th>Sum of Exterior Angles</th>
<th>One Interior Angle</th>
<th>One Exterior Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Each interior angle of a regular polygon is 135 degrees. How many sides does the polygon have? All work must be shown!
1) Name something (or 2 or 3 things...) that you found easy today

2) Name something (or 2 or 3 things...) that you found difficult today

3) Draw a pretty picture...
U2 D7: Test Review

Given: \( k \parallel \ell \) and \( m \parallel n \)

- a. Corresponding angles
- b. Alternate interior angles
- c. Alternate exterior angles
- d. Same-side interior angles
- e. Same-side exterior angles
- f. Vertical angles
- g. None of the above

Match each pair of angles with their name from the list above and tell if they are congruent (\( \equiv \)) or supplementary (S) or neither (N).

<table>
<thead>
<tr>
<th>ANGLES</th>
<th>NAME</th>
<th>( \equiv ), S, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 2 &amp; 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. 13 &amp; 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. 9 &amp; 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D. 11 &amp; 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. 4 &amp; 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F. 1 &amp; 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G. 15 &amp; 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H. 13 &amp; 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. 8 &amp; 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J. 7 &amp; 12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Find the value of $x$.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

13. 

14. 

15.
1. Complete the proof.

**Given:** ∠ADC and ∠AEB are right angles.

**Prove:** ∠ABE ≅ ∠ACD

<table>
<thead>
<tr>
<th>Proof</th>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>∠ADC is a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2.</td>
<td>∠AEB is a right angle.</td>
<td>2. Given</td>
</tr>
<tr>
<td>3.</td>
<td>∠ADC ≅ ∠AEB</td>
<td>3. ______</td>
</tr>
<tr>
<td>4.</td>
<td>∠A ≅ ∠A</td>
<td>4. ______</td>
</tr>
<tr>
<td>5.</td>
<td>∠ABE ≅ ∠ACD</td>
<td>5. ______</td>
</tr>
</tbody>
</table>

2. **Given:** \(\overline{AC} \parallel \overline{DE}\)

**Prove:** \(m\angle 3 = m\angle 1 + m\angle 2\)

3. **Given:** \(\overrightarrow{RT} \perp \overrightarrow{PQ}, \overrightarrow{WY} \perp \overrightarrow{PQ}\)

**Prove:** \(\angle 1 \equiv \angle 3\)
4. Given: \( \angle 1 \) is complementary to \( \angle 2 \).
Prove: \( PQ \perp PR \)

<table>
<thead>
<tr>
<th>Proof Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 ) is complementary to ( \angle 2 ).</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( m \angle 1 + m \angle 2 = 90 )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( m \angle 1 + m \angle 2 = m \angle QPR )</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ( \angle QPR ) is a right angle.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
</tr>
</tbody>
</table>

5. Given: \( \angle LMI \) is a right angle.
\( \angle 5 \) is complementary to \( \angle 1 \).
Prove: \( \angle 1 = \angle 6 \)

6. Given: \( \angle BCD = \angle CDE \), \( m \angle B + m \angle D = 180 \)
Prove: \( AB \parallel DC \)