# Unit 4 Syllabus: Properties of Triangles & Quadrilaterals

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1. Midsegment: A midsegment of a triangle is formed by connecting a segment between the midpoints of two of the sides of the triangle

a. Triangle Midsegment Theorem: The midsegment is parallel to the third side of the triangle, and it is equal to half the length.

\[ \overline{DE} \parallel \overline{AC} \]

If \( \overline{DE} = 5 \), then \( \overline{AC} = 10 \)

b. Each triangle can make three midsegments. It might help to redraw the triangle to see which pieces are parallel, and which piece is half of what...

\[ \overline{DF} \parallel \overline{AB} \]

\[ \overline{AC} \parallel \overline{EF} \]

\[ \overline{BC} \parallel \overline{DE} \]

a) \( \overline{AB} \parallel \______ \)  b) \( \overline{AB} = 8 \), so \( \overline{EF} = ____ \)

c) \( \overline{DF} \parallel ____ \)  d) \( \overline{DF} = 6 \), so \( \overline{BC} = ____ \)

e) \( \overline{AC} \parallel ____ \)  f) \( \overline{AC} = 10 \), so \( \overline{DE} = ____ \)

Answers: a) \( \overline{FE} \)  b) \( \overline{EF} = 4 \)  c) \( \overline{BC} \)  d) \( \overline{BC} = 12 \)  e) \( \overline{DE} \)  f) \( \overline{DE} = 5 \)
2. Three or more lines (rays, or segments) that intersect at a point are called concurrent—the point is called the “point of concurrency”

![Diagram of concurrent lines](image1)

**Concurrent Lines**
They all intersect at 1 point

**NonConcurrent Lines**
They intersect at multiple points

3. If a point is on the angle bisector, then the point is equidistant to the two rays of the angle
   a. The converse of this statement is also true

   ![Diagram of angle bisector](image2)

   *Note: The distance must always be at a right angle (perpendicular).
   *No matter where point D is on the bisector, the distances are always equal.

4. When we have a segment, and point on that segments perpendicular bisector is equidistant to the two endpoints of that segment...

   The 2 grey lines are equal!
   Follows from SAS \(\rightarrow\) CPCTC

![Diagram of perpendicular bisector](image3)

5. You can find a perpendicular bisector on the coordinate plane.
   a. First, find the midpoint of the segment
   b. Second, find the line that is perpendicular to the segment.
      i. If the line is horizontal \(\rightarrow\) use a vertical line
      ii. If the line is vertical \(\rightarrow\) use a horizontal line

   The endpoints of my grey segment are (1, 3) and (5, 3)
   So the midpoint is (3,3).
   Since the line is horizontal, I need to draw a vertical line...

![Coordinate plane](image4)

6. Perpendicular Bisector of a Triangle: A line that intersects a side of a triangle in two ways
   a. Perpendicular to the side
   b. At the midpoint of the side.

![Diagram of perpendicular bisector in a triangle](image5)
7. The 3 perpendicular bisectors are concurrent. The point where they intersect is called the circumcenter of the triangle.

![Circumcenter Diagram]

Notice how the three vertices of the triangle are on the circle. That means that the circumcenter is equidistant from the 3 vertices of the triangle.

You may be asked to find the circumcenter of a triangle on the coordinate plane.

Find the midpoints of the vertical and horizontal segments. Draw in the perpendicular lines. Their intersection is the circumcenter.

* You only need to find 2 because the other is concurrent!

8. The 3 angle bisectors of a triangle are concurrent. The point where they intersect is called the incenter of the triangle.

![Incenter Diagram]

The incenter is the center of a circle that can be inscribed in the triangle (drawn inside).

The distance from the incenter to each of the 3 sides of the triangle is equal (the thick lines shown above are all congruent – and perpendicular to the sides).
9. Median: A segment connecting a vertex of a triangle to the midpoint of the opposite side.

10. The 3 medians connect to form the centroid of the circle.
   a. The centroid is also called the center of gravity because triangular shaped objects will balance if an object is centered at the centroid.

11. Altitude: A segment from the vertex of a triangle to the opposite side such that the segment and the side are perpendicular. This determines the height of the triangle.

12. The 3 altitudes connect to make the orthocenter.
13. Triangles have two important properties:

**Property #1:** The smallest angle is across from the smallest side (S for Smallest)
  The medium angle is across from the medium side (M for Medium)
  The largest angle is across from the largest side (L for Largest)

**There is no formula to find the side lengths actual measures – you just compare them!**

List the angles in of the triangle shown in order from least to greatest.

**Answer**

List the angles in of the triangle shown in order from least to greatest.

**Property #2:** The two smallest sides of a triangle must add up to be larger than the largest

**The two smallest sides are 6 and 7.**

\[6 + 7 = 13 \rightarrow 13 \text{ is bigger than 12 (the largest side)}\]

So this is a triangle

**The two smallest sides are 4 and 5 \rightarrow they add up to 9.**

9 is **NOT** bigger than 14 (the largest side)

So this is **NOT** a triangle

***The sum must be **GREATER THAN**, not **EQUAL TO**.***

14. Can the following side measures be made to form a triangle?

- a. 6, 9, 13
- b. 4, 8, 12
- c. 15, 8, 31
- d. 6, 14, 15
- e. 10, 10, 8
- f. 4, 2, 5

Answers: yes, no, no, yes, yes, yes
### Summary of Triangle Segments

<table>
<thead>
<tr>
<th>Name of Segment</th>
<th>Altitude</th>
<th>Median</th>
<th>Perpendicular Bisector</th>
<th>Angle Bisector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture of the triangle with its segment</td>
<td><img src="image1.png" alt="Altitude" /></td>
<td><img src="image2.png" alt="Median" /></td>
<td><img src="image3.png" alt="Perpendicular Bisector" /></td>
<td><img src="image4.png" alt="Angle Bisector" /></td>
</tr>
<tr>
<td>Description</td>
<td>A perpendicular segment from one vertex to the opposite side</td>
<td>A segment from one vertex to the midpoint of the opposite side</td>
<td>A segment through the midpoint of a side that is also perpendicular to that side</td>
<td>Segment from a vertex to an opposite side such that the vertex is bisected into 2 congruent angles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name of Point of Concurrency</th>
<th>Orthocenter</th>
<th>Centroid (center of gravity)</th>
<th>Circumcenter</th>
<th>Incenter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture of all 3 segments drawn in, as well as the point of concurrency</td>
<td><img src="image5.png" alt="Orthocenter" /></td>
<td><img src="image6.png" alt="Centroid" /></td>
<td><img src="image7.png" alt="Circumcenter" /></td>
<td><img src="image8.png" alt="Incenter" /></td>
</tr>
<tr>
<td>Special Note(s)</td>
<td>The altitude can be inside, outside, or a side of the triangle</td>
<td>Median is broken up like this...</td>
<td>Makes a circle around the triangle... circumcenter is equidistant to the 3 vertices of the triangle</td>
<td>Makes a circle inside the triangle... incenter is equidistant from the 3 sides of the triangle (at right angles)</td>
</tr>
</tbody>
</table>

*Note: you do not need to memorize this entire chart – check with your teacher for the most important information!*

View the following website for more information about triangle centers. There are some nice interactive applets that allow you to really visualize the different segments and point.

To use the applet, click on the picture. Then click and drag one of the points in the picture.

http://www.math.sunysb.edu/~scott/mat360.spr04/cindy/Various.html
15. You probably want to read pages 280-283 for additional information.

16. (Reminders) A conditional is an if-then statement.
   a. Example: If it is raining, then I will bring an umbrella.
   b. Each conditional is made up of two parts:
      i. The hypothesis – the clause after the word “if.”
      ii. The conclusion – the clause after the word “then.”

   If an angle is between 0-90°, then it is a right angle.
   \[ \text{hypothesis} \quad \text{conclusion} \]

17. The converse of a conditional is created by switching the hypothesis and conclusion.

   **Conditional:** If an angle’s measure is between 0-90°, then it is a right angle.
   **Converse:** If it is a right angle, then its measure is between 0-90°.

   (just switch the two to create the converse)

18. The hypothesis and conclusion are both made up of statements.

19. The **negation** of a statement has the opposite truth value. (Add the word “not.”)
   a. Statement: \( x \) equals 8
   b. Negation: \( x \) does not equal 8.

   * The symbolic form of a negation is ~p, which is read “not p.” (p is the statement)

20. Two new words: **inverse** and **contrapositive**.
   a. Inverse: take the negation of both the hypothesis and conclusion for the *conditional*.
   b. Contrapositive: take the negation of both for the *converse*.

21. Example:
   a. Conditional: If it is raining, then I bring an umbrella
   b. Inverse: If it is not raining, then I do not bring my umbrella.
   c. Converse: If I bring my umbrella, then it is raining.
   d. Contrapositive: If I do not bring my umbrella, then it is not raining.
22. Equivalent statements are statements that have the same truth value.
   a. A conditional and its contrapositive **always** have the same truth value.
      i. Try it!
      ii. This idea is very helpful for **indirect reasoning**.

23. Indirect reasoning occurs when you don’t figure something out directly, but you eliminate all other possibilities, so you are confident something has to happen.
   a. You dad comes home from work with 3 candy bars. Snickers, Kit-Kat, Butterfinger.
   b. He tells you that you, your brother and your sister can each have one.
      i. You see your brother has Snickers.
      ii. You see your sister has Butterfinger
         1. You conclude INDIRECTLY that you will get the Kit-Kat. You don’t directly know that you get the Kit-Kat (if your Dad handed it to you), but you know indirectly because it’s the only one left.
         ** You proved that you MUST get the Kit-Kat because you can’t get the Snickers or Butterfinger.

24. This is a very simple idea of indirect reasoning. In math, sometimes it is easier to prove something indirectly, instead of directly. In other words, you prove that something can’t happen, meaning something else must happen.

25. If we want to prove a statement is true, sometimes all we need to do is prove that the negation is false!

26. To make an indirect proof, you need to find a contradiction.
   a. Example:
      i. Statement #1: \( \angle A \) is acute
      ii. Statement #2: \( \angle A \) is obtuse
         \[
         \text{These Statement contradict each other}
         \]

27. To write an indirect proof...
   a. State as an assumption the negation of what you are trying to prove
   b. Show that this assumption leads to a contradiction
   c. Conclude that the assumption must be false – therefore what you want to prove must be true!
Directions: Write the negation of each statement below

1) You are not sixteen years old.

2) The soccer game is on Friday.

Directions: use the conditional below to write the inverse and the contrapositive.

Conditional: If you like the Eagles, then you live in Philadelphia.

Inverse:

Contrapositive:

Directions: Determine if the two statements contradict each other or not

3) Statement #1: \( m\angle A = 90 \)
   Statement #2: \( m\angle A = 60 \)

4) Statement #1: Sally does not go to soccer practice on the weekends.
   Statement #2: Sally went to soccer practice on Thursday.

5) Statement #1: \( \overline{AB} \cong \overline{XY} \)
   Statement #2: \( \overline{AB} \perp \overline{XY} \)
Example

Given: \( \angle A \) and \( \angle B \) are not complementary.
Prove: \( \angle C \) is not a right angle.

Step 1: Assume that \( \angle C \) is a right angle.
Step 2: If \( \angle C \) is a right angle, then by the Triangle Angle-Sum Theorem
\[ m\angle A + m\angle B + 90 = 180. \]
So \( m\angle A + m\angle B = 90. \) Therefore, \( \angle A \)
and \( \angle B \) are complementary. But \( \angle A \) and \( \angle B \) are not complementary.
Step 3: Therefore, \( \angle C \) is not a right angle.

Exercises

Complete the proofs.

1. Arrange the statements given at the right to complete the steps of
the indirect proof.
Given: \( \overline{XY} \neq \overline{YZ} \)
Prove: \( \angle 1 \neq \angle 4 \)

Step 1: __?
Step 2: __?
Step 3: __?
Step 4: __?
Step 5: __?
Step 6: __?

A. But \( \overline{XY} \neq \overline{YZ} \).
B. Assume \( \angle 1 \equiv \angle 4 \).
C. Therefore, \( \angle 1 \neq \angle 4 \).
D. \( \angle 1 \) and \( \angle 2 \) are supplementary, and
\( \angle 3 \) and \( \angle 4 \) are supplementary.
E. According to the Converse of the Isosceles
Triangle Theorem, \( \overline{XY} = \overline{YZ} \) or \( \overline{XY} \equiv \overline{YZ} \).
F. If \( \angle 1 \equiv \angle 4 \), then by the Congruent Supplements
Theorem, \( \angle 2 \equiv \angle 3 \).

2. Complete the steps below to write a convincing argument using
indirect reasoning.
Given: \( \triangle DEF \) with \( \angle D \neq \angle F \)
Prove: \( \overline{EF} \neq \overline{DE} \)

Step 1: __?
Step 2: __?
Step 3: __?
Step 4: __?
Directions: From appearance, determine the best name for each figure.

3. 

4. 

5. 

6. 

7. 

8. 

Directions: Find the value of the variables then find the length of each side.

9. rhombus $ABDC$

10. parallelogram $LONM$

11. square $FGHI$
**Definition:** Opposite sides of a quadrilateral are parallel.

**Thm:** Opposite sides of a parallelogram are congruent.

**Thm:** Opposite angles of a parallelogram are congruent.

**Thm:** Consecutive angles of a parallelogram are supplementary.

**Thm:** The diagonals of a parallelogram bisect each other.
Directions: Solve for $x$ in each figure below.

10
\[ 2x - 4 \]

4x - 10
\[ 3x - 2 \]

Directions: Use the thm. Stating “if three (or more) parallel lines cut congruent segments of one transversal, then they cut congruent segments of every transversal (p. 315)” to find values below.

If $AE = 17$ and $BF = 18$, find the measures of the sides of parallelogram $BNXL$.

9. $BN$
10. $NX$
11. $XL$
12. $BL$

Directions: Find the value of the missing angles.

13. 
\[ \begin{array}{c}
1 \\
80^\circ
\end{array} \]

14. 
\[ \begin{array}{c}
1 \\
2 \\
140^\circ \\
3
\end{array} \]

19. 
\[ \begin{array}{c}
1 \\
2 \\
40^\circ \\
3 \\
47^\circ \\
2 \\
1 \\
3 \\
72^\circ \\
4 \\
50^\circ \\
2 \\
82^\circ \\
30^\circ
\end{array} \]

1) What is the definition of a parallelogram?

2) Name the 4 properties that you learned today about parallelograms.

3) What properties must hold to create a rhombus, square, rectangle, kite, and trapezoid?
There are 6 ways to prove you have a parallelogram

28. The definition (prove opposite sides are ________________)
29. Prove ________ pair(s) of opposite sides are _____________.
30. Prove ________ pair(s) of opposite angles are _____________.
31. Prove all pairs of consecutive angles are _________________.
32. Prove the diagonals _____________________________.
33. Prove that one pair of opposite sides are BOTH ___________ and ___________.

There are 3 stations around the room...

Station #1: Proofs
Station #2: Always, sometimes, or never
Station #3: Solve for x and y in the parallelogram

Wrap Up:

1) Name the 6 ways to prove a quadrilateral is a parallelogram
2) What has to be true for a quadrilateral to ...
   a. Always
   b. Sometimes
   c. Never
   ... be a parallelogram
34. Define the following terms (give the definition, not its properties):
   a. Parallelogram:

   b. Rhombus:

   c. Rectangle:

   d. Square:

35. The Rhombus:
   a. Definition –
   b. Property #1
   c. Property #2

36. The Rectangle:
   a. Definition –
   b. Property #1

37. The Square:
   a. Definition –
   b. Properties:
38. Rhombus:

a. If one diagonal of a parallelogram bisects two angles of the parallelogram, then it is a rhombus.

b. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

39. Rectangle:

a. If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle

Proving "Specials" on Coordinate Plane

In order to prove that a quadrilateral is special (parallelogram, rectangle, rhombus, square) you will need to prove one (or more) of 3 things.

1) Lines are parallel
2) Lines are perpendicular
3) Segments are congruent (equal in length)

To do so, you will need to use either the slope formula, or the distance formula

\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

\[m = \frac{y_2 - y_1}{x_2 - x_1}\]

= slopes ⇒ parallel lines
negative reciprocal slopes ⇒ perpendicular lines
1) Plot the points A (2, 0), B(5, 0), C(8, 1), D(5, 1) to create quadrilateral ABCD.

2) Prove that ABCD is a parallelogram by the definition (opposite sides parallel).

3) Prove ABCD is a parallelogram again, but this time do it by using the property that opposite sides must be congruent (think distance formula!).

Problems using the properties of “specials”

Find the measures of the numbered angles in each rectangle.

1. 

2. 

3. 

Find the measures of the numbered angles in each rhombus.

4. 

5. 

6.
40. **Trapezoid**: A quadrilateral with exactly _____________ pair(s) of opposite sides parallel.
   a. Parallel sides are called _________________.
   b. Non parallel sides are called _________________.

There is a **median** of a trapezoid just like there is a **midsegment** of a triangle.

![Trapezoid Diagram]

**Formula:**

41. **Isosceles Trapezoid**: A trapezoid with congruent __________________.

   Property #1: Both pairs of base angles are __________________

   Property #2: The diagonals are ______________

![Isosceles Trapezoid Diagram]

42. **Kite**: A quadrilateral with __________ pairs of _______________ sides congruent.

   Property #1:

   Property #2: The diagonals create _______________ triangles.
Directions: Find the measures of angles 1 and 2.

\[ m\angle 1 = \underline{\underline{\angle 1}}^\circ \]

\[ m\angle 2 = \underline{\underline{\angle 2}}^\circ \]

Directions: Find the values of \( x \) and \( y \) in each figure below.

\[ x = \underline{\underline{x}} \]

\[ y = \underline{\underline{y}} \]

Wrap up: Place a check mark in each appropriate box.

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<td>Opposite sides are parallel</td>
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<td>Opposite angles are congruent</td>
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<td>A diagonal forms two congruent triangles</td>
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<td>Diagonals bisect each other</td>
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<td>Diagonals are perpendicular</td>
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<td>Diagonals are angle bisectors</td>
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<td>All angles are right angles</td>
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<td>All sides are congruent</td>
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</tbody>
</table>
Directions: Solve for $x$ in each figure below.

1. 
2. 
3. 
4. 

Directions: Find the measure of each angle in the trapezoid

5. 
6. 

7. One angle of an isosceles trapezoid has measure 48. Find the measures of the other angles.

8. Two congruent angles of an isosceles trapezoid have measures $5x - 17$ and $2x + 13$. Find the value of $x$ and then give the measure of each angle of the trapezoid.
Directions: Look at Example 2 below to help you answer questions 9-11.

**Example 2** In \( \triangle ABC, AX = XM = MB \) and \( AY = YN = NC \).

a. If \( XY = 6 \), and \( BC = 18 \), find \( MN \).

b. If \( XY = 12 \), find \( MN \).

**Solution**

a. \( \overline{MN} \) is the median of trap. \( BXYC \).
\[
MN = \frac{1}{2}(XY + BC) = \frac{1}{2}(6 + 18) = 12
\]

b. \( \overline{XY} \) joins the midpoints of two sides of \( \triangle AMN \).
\[
XY = \frac{1}{2}MN \quad 12 = \frac{1}{2}MN \quad 24 = MN
\]

Use the diagram in Example 2. Complete.

9. If \( XY = 9 \), then \( MN = ? \) and \( BC = ? \).

10. If \( MN = 32 \), then \( XY = ? \) and \( BC = ? \).

11. If \( XY = 8 \) and \( MN = x + 12 \), then \( x = ? \) and \( BC = ? \).

Directions: Complete the proof below.

12. Given: Isosceles trap. \( ABCD \);
\[
\overline{CD} \equiv \overline{CE}
\]
Prove: \( ABCE \) is a \( \square \).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons (Postulates, Theorems, Definitions)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>
**Homework: U4D9, Review for Test**

Place a check in each box for the figure that has the listed property. Keep this check list as a reference for the entire unit.

<table>
<thead>
<tr>
<th>Properties of Quadrilaterals</th>
<th>Quadrilateral</th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
<th>Trapezoid</th>
<th>Isosceles Trapezoid</th>
<th>Right Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 sided polygon</td>
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<td>Sum of measures = 360</td>
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<td>One pair of parallel sides only</td>
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<td>Diagonals form congruent triangles</td>
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<td>Consecutive sides congruent</td>
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<td>One pair of opposite sides congruent only</td>
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<td>Diagonals bisect each other</td>
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<td>Diagonals bisect angles</td>
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<td>Consecutive angles supplementary</td>
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Proving a Quadrilateral is a Parallelogram

1. Prove both pairs of opposite sides parallel.
2. Prove both pairs of opposite sides congruent.
3. Prove one pair of opposite sides parallel and congruent.
4. Prove both pairs of opposite angles congruent.
5. Prove consecutive angles are supplementary.
6. Prove diagonals bisect each other.

Example

Given: $\overline{AE} \cong \overline{CE}$, $\overline{AD}$ is parallel to $\overline{BC}$
Prove: $ABCD$ is a parallelogram

Plan: Using method #6 above, if we show that the diagonals bisect each other, then we can prove the figure is a parallelogram. Since $\overline{AE} \cong \overline{CE}$ is given, we will only need to show that $\overline{DE} \cong \overline{BE}$.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. $\overline{AE} \cong \overline{CE}$, $\overline{AD}$ $\parallel \overline{BC}$</td>
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<tr>
<td>2. $\angle AED \cong \angle CEB$</td>
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<td>3. $\angle 3 \cong \angle 7$</td>
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<tr>
<td>4. $\triangle AED \cong \triangle CEB$</td>
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<tr>
<td>5. $\overline{DE} \cong \overline{BE}$</td>
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<tr>
<td>6. $ABCD$ is a parallelogram</td>
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</tbody>
</table>
Directions: Use the figure in the center to prove \#’s 5 and 6.

5. Given: triangle with $\overline{BD} \cong \overline{CD}$, $\overline{AE} \cong \overline{BD}$, and $\overline{AE} \parallel \overline{CD}$

Prove: $ACDF$ is a parallelogram.

6. Given: $\angle CBD \cong \angle C$, $\overline{AE} \cong \overline{BD}$, and $\overline{AC} \cong \overline{ED}$

Prove: $ACDE$ is a parallelogram.
Properties of the Rectangle, Rhombus, and Square

**Rectangle**
- all properties of parallelograms
  - all diagonals are congruent
  - all angles measure 90°

**Rhombus**
- all properties of parallelograms
  - all sides are congruent
  - all diagonals are perpendicular
  - all diagonals bisect opposite angles

**Square**
- all properties of parallelogram
- rectangle
- rhombus

Use the properties to solve for the missing measures in the diagrams.

LMNO is a rectangle. If LM = 16, MN = 12, and \( \angle 1 = 60° \), find the following:

a. \( ON = \) 

b. \( OL = \) 

e. \( \angle LON = \) 

f. \( \angle 2 = \) 

h. \( \angle 3 = \) 

i. \( \angle 4 = \) 

WXYZ is a rhombus. If WX = 4 and \( \angle WXY = 60° \), find the following:

a. \( XY = \) 

d. \( \angle 2 = \) 

b. \( \angle ZWX = \) 

e. \( \angle 3 = \) 

c. \( \angle 1 = \) 

f. \( \angle 4 = \) 

EFGH is a square. If EF = 10, find the following:

a. \( FG = \) 

g. \( \angle 1 = \) 

b. \( \angle EFG = \) 

h. \( \angle 3 = \) 

f. \( \angle EIF = \)
Plot each set of points to draw each figure

1. Is the parallelogram given by the points (-6,2) (2,6) (4,2) (-4,-2) a rectangle?

2. Is the parallelogram given by the points (-8,3) (0,9) (0,-1) (-8,-7) a rhombus?

3. Is the parallelogram given by the points (-6,4) (5,8) (9,-4) (-2,-8) a square?
Quadrilaterals

In Exercises 1–6 \(ABCD\) is a parallelogram. Complete.
1. If \(m\angle ADC = 92\), then \(m\angle ABC = \) _______ and \(m\angle DAB = \) _______.
2. If \(BD = 20\), then \(BE = \) _______.
3. If \(AB = 9x - 2\) and \(DC = 6x + 4\), then \(x = \) _______.
4. If \(AE = 9\) and \(AC = 5x + 3\), then \(x = \) _______.
5. If \(ABCD\) is a rectangle and \(DE = 13.4\), then \(AE = \) _______.
6. If \(ABCD\) is a rhombus, then \(m\angle AED = \) _______.

In Exercises 7–10 information is given about quadrilateral \(EFGH\). What additional information is needed to prove \(EFGH\) is a parallelogram?
7. \(\angle EHG \equiv \angle EFG\) _________________________________
8. \(EH \parallel FG\) _________________________________
9. \(EF \cong HG\) _________________________________
10. \(I\) is the midpoint of \(EG\). _________________________________

In Exercises 11–13, \(D\) is the midpoint of \(\overline{AB}\). Complete.
11. If \(E\) is the midpoint of \(\overline{BC}\) and \(AC = 26\), then \(\angle BDE \equiv \angle \) _______ and \(DE = \) _______.
12. If \(\overline{DE} \parallel \overline{AC}\) and \(BE = 12\), then \(BC = \) _______.
13. If \(\overline{BE} \cong \overline{EC}\), then \(ADEC\) is a(n) ________________.

In Exercises 14–17, \(M\) and \(N\) are the midpoints of \(\overline{TP}\) and \(\overline{RA}\), respectively, and \(TRAP\) is a trapezoid. Complete.
14. \(MN\) is the __________________ of \(TRAP\).
15. If \(\overline{TP} \cong \overline{RA}\), then \(TRAP\) is a(n) ________________.
16. If \(MN = 16\) and \(TR = 14\), then \(PA = \) _______.
17. If \(\overline{TP} \cong \overline{RA}\) and \(m\angle P = 80\), then \(m\angle A = \) _______ and \(m\angle TMN = \) _______.

Give the most descriptive name for quadrilateral \(QUAD\).
18. \(\overline{QU} \parallel \overline{DA}; \overline{OD} \parallel \overline{UA}; \overline{OD} \perp \overline{DA}\) ________________________________
19. \(\overline{QU} \parallel \overline{DA}; \overline{QU} \cong \overline{UA} \equiv \overline{DA}\) ________________________________
20. \(\overline{QU} \parallel \overline{DA}; \overline{OD} \equiv \overline{UA}; m\angle Q = m\angle U = 115\) ________________________________
Each diagram shows a trapezoid and its median. Find the value of \( x \).

1. \( \frac{22}{10} \)
   \[ x = \boxed{1} \]

2. \( \frac{x}{12} \)
   \[ x = \boxed{2} \]

3. \( \frac{x}{x + 2} \)
   \[ x = \boxed{3} \]

4. \( \frac{3x}{20} \)
   \[ x = \boxed{4} \]

5. \( \frac{5x}{7x} \)
   \[ x = \boxed{5} \]

6. \( \frac{18}{x^2} \)
   \[ x = \boxed{6} \]

In Exercises 7–12 \( \overline{EF} \) is the median of trapezoid \( ABCD \). Complete.

7. If \( m\angle A = 63 \), then \( m\angle DEF = \boxed{7} \) and 
   \( m\angle D = \boxed{8} \).

8. If \( m\angle CFE = 72 \), then \( m\angle B = \boxed{9} \) and 
   \( m\angle C = \boxed{10} \).

9. If \( AB = 16 \) and \( DC = 10 \), then \( EF = \boxed{11} \).

10. If \( AB = 21 \) and \( EF = 18 \), then \( DC = \boxed{12} \).

11. If \( ABCD \) is isosceles and \( m\angle B = 65 \), then \( m\angle A = \boxed{13} \), \( m\angle D = \boxed{14} \), 
    and \( m\angle C = \boxed{15} \).

12. If \( ABCD \) is isosceles, name all angles congruent to \( \angle A \).

In Exercises 13–17 \( \overline{IJ} = \overline{JL} = \overline{LG} \) and \( \overline{IK} = \overline{KM} = \overline{MH} \).

13. If \( JK = 5 \), then \( LM = \boxed{16} \) and \( GH = \boxed{17} \).

14. If \( LM = 12 \), then \( JK = \boxed{18} \) and \( GH = \boxed{19} \).

15. If \( JK = 10 \) and \( LM = x + 8 \), then \( x = \boxed{20} \).

16. If \( GH = 36 \), then \( LM = \boxed{21} \) and \( JK = \boxed{22} \).

17. If \( LM = 4x \) and \( GH = x + 6 \), write \( JK \) in terms of \( x \).
   \[ JK = \boxed{23} \]. Then \( x = \boxed{24} \).
Quadrilateral Review

Fill the blanks with ALWAYS, SOMETIMES, or NEVER to create a true statement.

1. A parallelogram is ___________ a quadrilateral.
2. A rectangle is ___________ a trapezoid.
3. A rhombus is ___________ a parallelogram.
4. A square is ___________ a quadrilateral.
5. A rectangle is ___________ a rhombus.
6. A rhombus is ___________ a square.
7. A trapezoid is ___________ isosceles.
8. A rectangle is ___________ a square.
9. A square is ___________ a rectangle.
10. A trapezoid is ___________ a parallelogram.
11. A rectangle ___________ has four right angles.
12. A rhombus ___________ has four right angles.
13. A quadrilateral is ___________ a pentagon.
14. A parallelogram is ___________ equilateral.
15. A trapezoid is ___________ equilateral.

1. Given: $MA \parallel HT$, $\angle 4 \equiv \angle 2$
   Prove: MATH is a parallelogram

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons (Postulates, Theorems, Definitions)</th>
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**Part I:**  **Directions** – make the statement true using *always, sometimes, or never.*

43. The diagonals of a square are perpendicular. ________________
44. A rhombus has congruent diagonals. ________________
45. If a parallelogram’s diagonals bisect the angles, then the figure ________________ is a rhombus.
46. In a trapezoid, there are two pairs of supplementary angles. ________________
47. If $EFGH$ is a rectangle, then $EFGH$ is also a square. ________________
48. All consecutive angles of rhombus are supplementary. ________________
49. A square has four congruent angles. ________________

**Part II:**  **Directions** – Use the rhombus below to solve for $x$ and $y$.

```
 2x + 4
(x + 8)°
---
3y + 8

(4y + 5)°
```